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UPPER PERCENTAGE POINTS OF THE INDIVIDUAL ROOTS OF THE WISHART MATRIX

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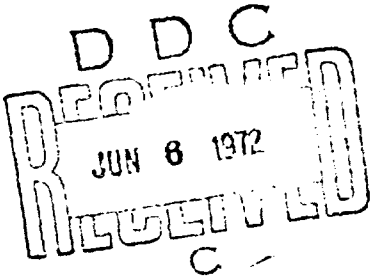

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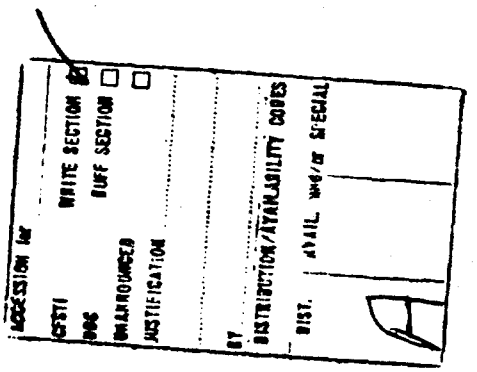

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Let S: pxp be distributed as central Wishart matrix with n degrees of freedom and let $E(S) = n I_p$ where I_p is an identity matrix. Also, let $\theta_1 < \ldots < \theta_p$ be the roots of S. In this report, the authors gave exact upper 10%, 5%, 2.5% and 1% points of the distributions of θ_i (i=1,2,..., p-1) for p=2(1)10 and n=(p+1)(1)20(2)30(5)50.

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AEROSPACE RESEARCH LABORATORIES
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FOREWORD

This report was prepared for the Applied Mathematics Research Laboratory, Aerospace Research Laboratories by D. S. Clemm, A. K. Chattopadhyay and P. R. Krishnaiah under Project 7071, "Research in Applied Mathematics". The work of A. K. Chattopadhyay was performed at the Aerospace Research Laboratories while in the capacity of Technology Incorporated Visiting Research Associate under Contract F 33615-71-C-1463, T. I. Project No. 4262B.

In this report, the authors gave exact percentage points of the smallest and intermediate roots of the Wishart matrix.

The authors wish to thank Dr. V. B. Waikar for some helpful discussions. They also wish to thank Miss Eva Brandenburg for typing the manuscript carefully.

ABSTRACT

Let S: pxp be distributed as central Wishart matrix with n degrees of freedom and let $E(S) = I_p$ where I_p is an identity matrix. Also, let $\theta_1 < \ldots < \theta_p$ be the roots of S. In this report, the authors gave <u>exact</u> upper 10%, 5%, 2.5% and 1% points of the distributions of θ_i (i=1,2,...,p-1) for p=2(1)10 and n=(p+1)(1)20 (2)30(5)50.

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1. INTRODUCTION

The marginal distribution of the individual roots of the Wishart matrix are useful in testing certain statistical hypotheses. Pillai and Chang [7] constructed tables for the upper percentage points of the largest root of the Wishart matrix. Davis [2] gave a recurrence relation for the marginal densities of the individual roots by using results in [1]. Krishnaiah and Waikar [5] gave exact expressions for the cumulative distribution functions (c.d.f.'s) of intermediate roots. In this paper, we give exact upper 10%, 5%, 2.5% and 1% points of the smallest and intermediate roots of the Wishart matrix.

2. CUMULATIVE DISTRIBUTION FUNCTIONS OF THE INDIVIDUAL ROOTS

Let S:pxp be distributed as Wishart matrix with n degrees of freedom and let $E(S) = n\, I_p$, where I_p is the pth order identity matrix. Also, let $\theta_1<\ldots<\theta_p$ be the eigenvalues of S. Then the joint density of $\theta_1<\ldots<\theta_p$ is given by

$$f(\theta_1,\ldots,\theta_p) = k(p,n) \prod_{i=1}^{p} [\theta_i^r \exp(-\frac{\theta_i}{2})] \prod_{i>j} (\theta_i-\theta_j)$$

$$0<\theta_1<\ldots<\theta_p<\infty \qquad (2.1)$$

where

$$r = (n-p-1)/2, \qquad \text{and}$$

$$k(p,n) = \pi^{p/2} (\frac{1}{2})^{\frac{np}{2}} / \prod_{i=1}^{p} [\Gamma((n+1-i)/2)\Gamma((p+1-i)/2)], \qquad (2.1a).$$

The following exact expression for the c.d.f. of the intermediate root θ_s $(1<s<p-1)$ was given in Krishnaiah and Waikar [5]:

$$P[\theta_s<x] = P[\theta_{s+1}<x] + k(p,n)\sum_{1}\pm\rho(\psi;s,\{k_1,\ldots,k_s\}, 0,x).$$

$$\rho(\psi;\ p-s,\ \{t_1,\ldots,\ t_{n-s}\},\ x,\infty), \qquad (2.2)$$

where $\{k_1,\ldots,k_s\}$ is a subset of integers $\{0,1,\ldots,p-1\}$ such that $k_1<\ldots<k_s$ and $t_1<\ldots<t_{p-s}$ is the subset complementary to $k_1<\ldots<k_s$ while $\sum_1$ denotes the summation over $\binom{p}{s}$ possible subsets $k_1<\ldots<k_s$. Further $\psi(y) = \exp\left(-\frac{y}{2}\right)$ and the sign inside $\sum_1$ is positive or negative according as $s(s+3)/2 + \sum_{i=1}^{s} k_i$ is even or odd. The function $\rho(\cdot)$ is defined by

$$\rho(\psi;\ p,\{k_1,\ldots,k_p\},\ L,U) = \Delta(\psi;\ 2m,\{k_1,\ldots,k_{2m}\},L,U) \text{ when } p = 2m \qquad (2.3)$$

and

$$\rho(\psi;\ p,\{k_1,\ldots,k_p\},\ L,U) = \sum_{i=1}^{2m+1} (-1)^{i+1}\ F_{k_i}(L,U).$$

$$G_i(\psi;\ 2m+1,\ \{k_1,\ldots,k_{2m+1}\},\ L,U) \quad \text{when } p = 2m+1 \qquad (2.4)$$

where

$$\Delta(\psi;\ 2m,\{k_1,\ldots,k_{2m}\},\ L,U) = \left|\left(f_{k_i}^{k_j}(L,U)\right)\ i,j=1,\ ,2m\right|^{1/2},$$

$$G_t(\psi;\ 2m+1,\{k_1,\ldots,\ k_{2m+1}\},\ L,U)$$

$$=\left|\left(f_{k_i}^{k_j}(L,U)\right)\ i,j=1,\ldots,\ t-1,\ t+1,\ldots,2m+1\right|^{1/2}$$

for $t=1,\ldots,2m+1$ while $G_1(\psi;\ 1,k_1,L,U)=1$. Further

$$f_s^t\ (L,U)= F_s^t\ (L,U)-F_t^s(L,U),$$

$$F_s^t(L,U) = \int_L^U F_s(L,\theta)\ \theta^t\ \psi(\theta)\ d\theta,$$

$$F_s(L,\theta) = \int_L^\theta x^s\ \psi(x)\ dx\ .$$

Also it is known (see[3,4]),that

$$P[\theta_1\leq x] = 1-k(p,n)\rho(\psi;1,r,c,\infty). \qquad (2.5)$$

Using (2.2) and (2.5), we have constructed tables for the exact values of x for $p=2(1)10$, $n=(p+1)(1)20(2)30(5)50$, $k=1(1)(p-1)$ and $\alpha=0.01,\ 0.025,\ 0.05,\ 0.10$ where

$$P[\theta_s \leq x] = 1-\alpha.$$

The values are given up to four decimal places and they may differ from actual values by at most one unit in the last decimal. As a check for the accuracy of the tables we have used the programs to compute the values of

$$P[x\leq\theta_1] + P[\theta_1\leq x\leq\theta_2] + \ldots + P[\theta_{p-1}\leq x\leq\theta_p] + P[\theta_p\leq x]$$

for some x and found them to differ from 1 in the 12th decimal only.

REFERENCES

[1] Davis, A. W. (1970). On the marginal distributions of the latent roots of the multivariate beta matrix. Mimeo Series No. 690, Institute of Statistics, University of North Carolina.

[2] Davis, A. W. (1971). On the distributions of the latent roots and traces of certain random matrices. (to appear) J. Multivariate Analysis.

[3] de Bruijn, N. G. (1955). On some multiple integrals involving determinants. J. Indian Math. Soc. 19, 133-152.

[4] Krishnaiah, P. R. and Chang, T. C. (1971). On the exact distributions of the extreme roots of the Wishart and MANOVA matrices. J. Multivariate Analysis. 1, 108-117.

[5] Krishnaiah, P. R. and Waikar, V. B. (1971). Exact joint distributions of any few ordered roots of a class of random matrices. J. Multivariate Analysis. 1, 308-315.

[6] Mehta, M. L. (1960). On the statistical properties of the level spacings in nuclear spectra. Nucl. Phys., 18, 395-419.

[7] Pillai, K.C.S. and Chang, T. C. (1970). An approximation to the c.d.f. of the largest root of a covariance matrix. Ann. Inst. Statist. Math. Supplement 6, 115-124.

The entries in the following table are the values of x for different values of N,P,S and α where

$$P[\theta_S \leq x] = (1-\alpha),$$

$$f(\theta_1,\ldots,\theta_p) = k(P,N) \prod_{i=1}^{p} [\theta_i^r \exp(-\theta_i/2)] \prod_{i>j} (\theta_i-\theta_j)$$

$$0<\theta_1<\ldots<\theta_S<\ldots<\theta_p<\infty ,$$

$r = (N-P-1)/2$, and $k(P,N)$ is given by Eq. (2.1a).

TABLE 1

UPPER PERCENTAGE POINTS OF THE INDIVIDUAL ROOTS

P= 2 S=1

N \ α	.01	.025	.05	.10
3	4.61	3.69	3.00	2.30
4	6.09	5.04	4.23	3.40
5	7.49	6.33	5.42	4.48
6	8.83	7.58	6.59	5.55
7	10.15	8.81	7.74	6.61
8	11.44	10.01	8.88	7.67
9	12.70	11.21	10.01	8.72
10	13.95	12.39	11.12	9.75
11	15.19	13.56	12.24	10.81
12	16.41	14.72	13.34	11.85
13	17.63	15.87	14.45	12.89
14	18.83	17.02	15.54	13.93
15	20.03	18.16	16.64	14.97
16	21.22	19.30	17.73	16.00
17	22.41	20.43	18.81	17.04
18	23.58	21.56	19.90	18.07
19	24.76	22.68	20.98	19.11
20	25.93	23.81	22.06	20.14
22	28.25	26.04	24.21	22.20
24	30.56	28.26	26.36	24.25
26	32.85	30.47	28.50	26.31
28	35.14	32.67	30.63	28.35
30	37.41	34.87	32.76	30.41
35	43.05	40.33	38.06	35.53
40	48.65	45.76	43.35	40.64
45	54.21	51.16	48.61	45.75
50	59.74	56.54	53.86	50.84

P= 3 S=1

N \ α	.01	.025	.05	.10
4	3.07	2.46	2.00	1.54
5	4.21	3.49	2.94	2.36
6	5.33	4.51	3.88	3.21
7	6.42	5.53	4.82	4.07
8	7.51	6.54	5.76	4.94
9	8.59	7.55	6.71	5.82
10	9.66	8.55	7.66	6.70
11	10.72	9.56	8.62	7.59
12	11.79	10.56	9.57	8.49
13	12.84	11.57	10.53	9.39
14	13.90	12.57	11.49	10.30
15	14.95	13.58	12.44	11.20
16	16.01	14.58	13.41	12.12
17	17.05	15.58	14.37	13.03
18	18.10	16.58	15.33	13.95
19	19.15	17.59	16.29	14.86
20	20.19	18.59	17.26	15.79
22	22.28	20.59	19.19	17.63
24	24.36	22.59	21.12	19.49
26	26.44	24.60	23.06	21.35
28	28.51	26.60	25.00	23.22
30	30.58	28.60	26.94	25.09
35	35.75	33.60	31.80	29.78
40	40.90	38.61	36.68	34.50
45	46.05	43.61	41.55	39.23
50	51.18	48.61	46.44	43.98

TABLE 1 (CONTINUED)

P= 3 S =2

N	α .01	.025	.05	.10
4	8.41	7.22	6.30	5.32
5	10.05	8.77	7.75	6.68
6	11.60	10.24	9.15	7.99
7	13.11	11.67	10.51	9.27
8	14.57	13.06	11.84	10.53
9	16.00	14.42	13.15	11.77
10	17.40	15.76	14.43	12.99
11	18.78	17.08	15.71	14.21
12	20.14	18.39	16.96	15.41
13	21.49	19.68	18.21	16.60
14	22.82	20.96	19.44	17.78
15	24.14	22.23	20.67	18.95
16	25.45	23.49	21.89	20.13
17	26.74	24.74	23.10	21.29
18	28.03	25.98	24.30	22.45
19	29.31	27.22	25.50	23.61
20	30.58	28.45	26.69	24.76
22	33.10	30.89	29.06	27.05
24	35.60	33.31	31.41	29.32
26	38.08	35.71	33.75	31.58
28	40.53	38.10	36.08	33.84
30	42.98	40.47	38.39	36.08
35	49.01	46.34	44.13	41.65
40	54.98	52.16	49.81	47.19
45	60.88	57.92	55.45	52.69
50	65.74	63.64	61.05	58.16

P= 4 S=1

N	α .01	.025	.05	.10
5	2.30	1.84	1.50	1.15
6	3.23	2.68	2.26	1.82
7	4.16	3.53	3.04	2.52
8	5.09	4.39	3.83	3.24
9	6.02	5.25	4.64	3.98
10	6.96	6.13	5.46	4.74
11	7.89	7.01	6.29	5.51
12	8.83	7.89	7.12	6.29
13	9.77	8.78	7.97	7.08
14	10.71	9.67	8.82	7.88
15	11.66	10.57	9.67	8.69
16	12.60	11.47	10.54	9.51
17	13.55	12.37	11.40	10.33
18	14.50	13.28	12.27	11.15
19	15.45	14.19	13.14	11.93
20	16.40	15.10	14.02	12.82
22	18.31	16.93	15.78	14.50
24	20.22	18.77	17.56	16.20
26	22.13	20.61	19.34	17.92
28	24.05	22.46	21.13	19.64
30	25.97	24.31	22.93	21.37
35	30.78	28.97	27.45	25.74
40	35.60	33.65	32.01	30.16
45	40.43	38.36	36.60	34.61
50	45.28	43.07	41.21	39.09

TABLE 1 (CONTINUED)

P= 4 S =2

N	α .01	.025	.05	.10
5	5.03	5.20	4.54	3.85
6	7.37	6.46	5.73	4.95
7	8.67	7.58	6.89	6.04
8	9.94	8.88	8.03	7.11
9	11.19	10.07	9.16	8.13
10	12.42	11.24	10.28	9.24
11	13.63	12.40	11.40	10.39
12	14.84	13.55	12.51	11.35
13	15.03	14.70	13.61	12.41
14	17.22	15.84	14.70	13.46
15	18.39	16.97	15.80	14.51
16	19.57	18.10	16.89	15.55
17	20.73	19.22	17.97	16.60
18	21.89	20.34	19.06	17.64
19	23.04	21.45	20.14	18.68
20	24.19	22.56	21.22	19.72
22	26.48	24.78	23.37	21.80
24	28.76	26.98	25.51	23.87
26	31.02	29.18	27.65	25.94
28	33.27	31.36	29.78	28.01
30	35.51	33.54	31.90	30.07
35	41.08	38.97	37.20	35.22
40	46.60	44.36	42.47	40.35
45	52.10	49.72	47.73	45.49
50	57.56	55.07	52.97	50.61

P= 4 S =3

N	α .01	.025	.05	.10
5	12.21	10.83	9.74	8.56
6	13.92	12.47	11.31	10.05
7	15.57	14.05	12.82	11.49
8	17.16	15.57	14.29	12.90
9	18.71	17.06	15.73	14.27
10	20.23	18.52	17.13	15.62
11	21.72	19.96	18.52	16.96
12	23.18	21.37	19.89	18.27
13	24.62	22.76	21.24	19.57
14	26.05	24.14	22.57	20.86
15	27.46	25.50	23.90	22.14
16	28.85	26.85	25.21	23.40
17	30.23	28.19	26.51	24.66
18	31.60	29.52	27.80	25.91
19	32.96	30.83	29.08	27.15
20	34.31	32.14	30.36	28.39
22	36.98	34.74	32.89	30.84
24	39.62	37.30	35.39	33.27
26	42.23	39.85	37.87	35.69
28	44.82	42.37	40.34	38.08
30	47.38	44.87	42.78	40.47
35	53.72	51.05	48.83	46.37
40	59.96	57.15	54.81	52.20
45	66.12	63.18	60.73	57.99
50	72.21	69.15	66.59	63.73

TABLE 1 (CONTINUED)

P= 5 S=1

N	α .01	.025	.05	.10
6	1.84	1.48	1.20	.92
7	2.63	2.16	1.84	1.43
8	3.42	2.91	2.50	2.08
9	4.23	3.65	3.19	2.70
10	5.05	4.41	3.90	3.35
11	5.88	5.18	4.62	4.02
12	6.71	5.96	5.36	4.70
13	7.55	6.76	6.11	5.40
14	8.40	7.56	6.87	6.11
15	9.25	8.37	7.64	6.84
16	10.11	9.18	8.41	7.57
17	10.98	10.00	9.20	8.31
18	11.84	10.83	9.99	9.05
19	12.71	11.66	10.79	9.82
20	13.59	12.50	11.59	10.58
22	15.35	14.18	13.21	12.13
24	17.11	15.88	14.85	13.70
26	18.89	17.59	16.50	15.28
28	20.68	19.31	18.17	16.89
30	22.47	21.05	19.85	18.50
35	26.99	25.42	24.10	22.60
40	31.54	29.83	28.39	26.76
45	36.11	34.29	32.74	30.98
50	40.72	38.77	37.12	35.24

P= 5 S=2

N	α .01	.025	.05	.10
6	4.73	4.08	3.57	3.03
7	5.87	5.15	4.57	3.96
8	6.99	6.20	5.57	4.89
9	8.10	7.25	6.56	5.82
10	9.19	8.29	7.55	6.76
11	10.28	9.32	8.54	7.70
12	11.36	10.36	9.53	8.64
13	12.44	11.39	10.52	9.58
14	13.51	12.41	11.51	10.52
15	14.58	13.44	12.50	11.47
16	15.65	14.47	13.49	12.42
17	16.71	15.49	14.48	13.37
18	17.77	16.51	15.47	14.32
19	18.83	17.53	16.46	15.27
20	19.89	18.55	17.45	16.22
22	21.99	20.59	19.43	18.14
24	24.10	22.63	21.41	20.05
26	26.20	24.67	23.39	21.97
28	28.29	26.70	25.37	23.89
30	30.38	28.73	27.36	25.81
35	35.59	33.81	32.31	30.63
40	40.78	38.88	37.27	35.46
45	45.96	43.94	42.23	40.31
50	51.13	49.00	47.19	45.16

TABLE 1 (CONTINUED)

P= 5 S =3

N	α .01	.025	.05	.10
6	9.16	8.16	7.36	6.49
7	10.62	9.55	8.68	7.75
8	12.03	10.89	9.98	8.98
9	13.40	12.21	11.25	10.19
10	14.75	13.51	12.50	11.39
11	16.08	14.79	13.73	12.57
12	17.39	16.05	14.95	13.74
13	18.69	17.30	16.16	14.91
14	19.97	18.54	17.36	16.07
15	21.25	19.77	18.56	17.22
16	22.51	20.99	19.74	18.36
17	23.76	22.21	20.92	19.50
18	25.00	23.41	22.10	20.64
19	26.24	24.61	23.26	21.77
20	27.47	25.81	24.43	22.90
22	29.91	28.18	26.74	25.14
24	32.33	30.53	29.04	27.37
26	34.73	32.87	31.32	29.59
28	37.12	35.20	33.59	31.81
30	39.49	37.51	35.86	34.01
35	45.36	43.24	41.48	39.49
40	51.17	48.93	47.05	44.94
45	56.93	54.56	52.58	50.36
50	62.64	60.17	58.09	55.75

P= 5 S=4

N	α .01	.025	.05	.10
6	16.02	14.50	13.26	11.93
7	17.79	16.19	14.90	13.50
8	19.49	17.83	16.49	15.03
9	21.14	19.43	18.03	16.51
10	22.75	20.99	19.54	17.97
11	24.33	22.51	21.03	19.40
12	25.88	24.01	22.48	20.81
13	27.41	25.49	23.92	22.20
14	28.91	26.95	25.34	23.57
15	30.39	28.39	26.74	24.93
16	31.86	29.81	28.13	26.28
17	33.31	31.22	29.51	27.61
18	34.75	32.62	30.87	28.94
19	36.17	34.01	32.22	30.25
20	37.58	35.38	33.57	31.56
22	40.38	38.10	36.23	34.15
24	43.14	40.79	38.85	36.71
26	45.86	43.45	41.46	39.24
28	48.56	46.08	44.03	41.76
30	51.23	48.69	46.59	44.25
35	57.81	55.13	52.90	50.42
40	64.27	61.46	59.12	56.51
45	70.65	67.71	65.26	62.53
50	76.95	73.89	71.34	68.49

TABLE 1 (CONTINUED)

P= 6 S=1

N	α .01	.025	.05	.10
7	1.54	1.23	1.00	.77
8	2.21	1.84	1.55	1.25
9	2.91	2.47	2.13	1.77
10	3.63	3.13	2.74	2.32
11	4.36	3.81	3.37	2.90
12	5.10	4.50	4.02	3.49
13	5.85	5.21	4.68	4.11
14	6.62	5.93	5.36	4.75
15	7.40	6.66	6.06	5.40
16	8.18	7.40	6.76	6.06
17	8.97	8.15	7.47	6.73
18	9.76	8.90	8.19	7.41
19	10.56	9.67	8.93	8.10
20	11.37	10.44	9.66	8.81
22	13.00	11.99	11.16	10.23
24	14.64	13.57	12.68	11.68
26	16.30	15.17	14.22	13.16
28	17.98	16.79	15.78	14.66
30	19.66	18.41	17.36	16.17
35	23.92	22.53	21.36	20.03
40	28.24	26.72	25.43	23.98
45	32.59	30.95	29.56	7.98
50	36.98	35.23	33.74	32.05

P= 6 S=2

N	α .01	.025	.05	.10
7	3.90	3.37	2.95	2.50
8	4.90	4.30	3.82	3.31
9	5.88	5.22	4.69	4.12
10	6.86	6.15	5.57	4.95
11	7.84	7.08	6.46	5.78
12	8.82	8.01	7.35	6.62
13	9.80	8.94	8.24	7.47
14	10.77	9.88	9.14	8.33
15	11.75	10.81	10.04	9.19
16	12.73	11.75	10.95	10.05
17	13.71	12.69	11.85	10.92
18	14.69	13.63	12.76	11.80
19	15.67	14.58	13.68	12.67
20	16.65	15.52	14.59	13.55
22	18.61	17.42	16.43	15.32
24	20.57	19.31	18.27	17.10
26	22.53	21.22	20.12	18.89
28	24.49	23.12	21.98	20.69
30	26.45	25.03	23.84	22.50
35	31.37	29.81	28.51	27.04
40	36.29	34.61	33.20	31.61
45	41.21	39.42	37.92	36.21
50	46.13	44.24	42.65	40.84

TABLE 1 (CONTINUED)

P= 6 S=3

N \ α	.01	.025	.05	.10
7	7.40	6.61	5.98	5.27
8	8.57	7.81	7.12	6.35
9	9.91	9.00	8.25	7.44
10	11.13	10.16	9.37	8.51
11	12.34	11.32	10.49	9.58
12	13.53	12.47	11.60	10.64
13	14.71	13.61	12.70	11.69
14	15.88	14.74	13.79	12.75
15	17.05	15.86	14.88	13.80
16	18.21	16.98	15.97	14.85
17	19.36	18.10	17.05	15.89
18	20.51	19.21	18.13	16.94
19	21.65	20.32	19.21	17.98
20	22.79	21.42	20.29	19.02
22	25.05	23.62	22.43	21.10
24	27.30	25.81	24.57	23.18
26	29.54	27.99	26.70	25.25
28	31.77	30.16	28.82	27.31
30	33.99	32.33	30.94	29.38
35	39.51	37.72	36.22	34.53
40	44.99	43.08	41.48	39.67
45	50.44	48.42	46.72	44.80
50	55.86	53.74	51.95	49.93

P= 6 S=4

N \ α	.01	.025	.05	.10
7	12.41	11.27	10.35	9.35
8	13.94	12.74	11.77	10.70
9	15.42	14.17	13.15	12.03
10	16.88	15.58	14.51	13.34
11	18.30	16.95	15.84	14.63
12	19.70	18.31	17.16	15.90
13	21.09	19.65	18.46	17.16
14	22.45	20.97	19.75	18.40
15	23.80	22.28	21.03	19.64
16	25.14	23.58	22.29	20.87
17	26.47	24.87	23.55	22.09
18	27.78	26.15	24.80	23.30
19	29.09	27.42	26.04	24.50
20	30.39	28.68	27.27	25.70
22	32.96	31.19	29.72	28.09
24	35.50	33.67	32.15	30.45
26	38.02	36.13	34.56	32.80
28	40.51	38.57	36.95	35.13
30	42.99	40.99	39.32	37.46
35	49.12	46.99	45.21	43.21
40	55.16	52.91	51.03	48.91
45	61.14	58.78	56.80	54.57
50	67.07	64.60	62.52	60.19

TABLE 1 (CONTINUED)

P= 6 S=5

N	α .01	.025	.05	.10
7	19.88	18.20	16.85	15.38
8	21.65	19.94	18.54	17.01
9	23.39	21.62	20.18	18.60
10	25.09	23.27	21.77	20.14
11	26.75	24.87	23.34	21.65
12	28.37	26.45	24.88	23.15
13	29.97	28.00	26.39	24.62
14	31.54	29.53	27.88	26.07
15	33.09	31.04	29.35	27.50
16	34.62	32.53	30.81	28.91
17	36.13	34.00	32.25	30.31
18	37.63	35.46	33.67	31.70
19	39.11	36.91	35.09	33.08
20	40.58	38.34	36.49	34.45
22	43.48	41.17	39.26	37.15
24	46.34	43.97	42.00	39.82
26	49.17	46.73	44.71	42.47
28	51.96	49.46	47.39	45.08
30	54.72	52.16	50.04	47.68
35	61.52	58.82	56.58	54.08
40	68.18	65.36	63.00	60.38
45	74.75	71.80	69.34	66.60
50	81.23	78.16	75.61	72.75

P= 7 S =1

N	α .01	.025	.05	.10
8	1.32	1.05	.86	.66
9	1.91	1.59	1.34	1.08
10	2.54	2.16	1.85	1.54
11	3.18	2.75	2.40	2.03
12	3.84	3.36	2.97	2.55
13	4.51	3.99	3.56	3.10
14	5.20	4.63	4.17	3.66
15	5.91	5.29	4.79	4.24
16	6.62	5.96	5.43	4.84
17	7.34	6.65	6.07	5.45
18	8.07	7.34	6.73	6.07
19	8.81	8.04	7.41	6.70
20	9.56	8.75	8.09	7.35
22	11.07	10.20	9.47	8.66
24	12.60	11.67	10.89	10.01
26	14.16	13.16	12.33	11.39
28	15.73	14.68	13.79	12.79
30	17.32	16.21	15.27	14.22
35	21.35	20.11	19.05	17.86
40	25.45	24.08	22.92	21.51
45	29.61	28.13	26.86	25.43
50	33.82	32.22	30.86	29.32

TABLE 1 (CONTINUED)

P= 7 S=2

N	α .01	.025	.05	.10
8	3.33	2.87	2.51	2.14
9	4.20	3.69	3.28	2.84
10	5.08	4.52	4.06	3.57
11	5.97	5.35	4.85	4.31
12	6.85	6.19	5.65	5.07
13	7.75	7.04	6.46	5.83
14	8.64	7.89	7.28	6.61
15	9.54	8.75	8.10	7.39
16	10.44	9.61	8.93	8.19
17	11.34	10.48	9.77	8.98
18	12.25	11.35	10.61	9.78
19	13.16	12.23	11.45	10.59
20	14.07	13.10	12.30	11.41
22	15.90	14.87	14.01	13.06
24	17.73	16.64	15.73	14.72
26	19.58	18.43	17.47	16.40
28	21.42	20.22	19.22	18.09
30	23.28	22.02	20.97	19.79
35	27.93	26.55	25.39	24.08
40	32.51	31.12	29.86	28.43
45	37.31	35.71	34.35	32.82
50	42.02	40.32	38.88	37.24

P= 7 S=3

N	α .01	.025	.05	.10
8	6.24	5.57	5.03	4.45
9	7.36	6.64	6.05	5.42
10	8.47	7.70	7.07	6.38
11	9.57	8.75	8.08	7.34
12	10.66	9.80	9.09	8.31
13	11.75	10.84	10.09	9.27
14	12.83	11.88	11.10	10.23
15	13.90	12.91	12.10	11.20
16	14.97	13.95	13.10	12.16
17	16.04	14.98	14.10	13.13
18	17.11	16.01	15.10	14.10
19	18.17	17.04	16.11	15.06
20	19.23	18.07	17.11	16.03
22	21.35	20.13	19.11	17.97
24	23.46	22.18	21.11	19.91
26	25.56	24.22	23.11	21.85
28	27.66	26.27	25.11	23.80
30	29.75	28.31	27.10	25.75
35	34.98	33.41	32.10	30.62
40	40.18	38.50	37.09	35.50
45	45.37	43.59	42.09	40.39
50	50.55	48.67	47.08	45.29

TABLE 1 (CONTINUED)

P= 7 S=4

N \ α	.01	.025	.05	.10
8	10.24	9.32	8.57	7.76
9	11.60	10.63	9.83	8.96
10	12.93	11.90	11.06	10.14
11	14.23	13.16	12.28	11.31
12	15.52	14.40	13.49	12.48
13	16.79	15.63	14.68	13.63
14	18.05	16.85	15.86	14.77
15	19.29	18.06	17.04	15.91
16	20.53	19.26	18.21	17.04
17	21.75	20.45	19.37	18.17
18	22.98	21.64	20.53	19.29
19	24.19	22.82	21.68	20.41
20	25.40	24.00	22.83	21.52
22	27.80	26.33	25.11	23.75
24	30.17	28.65	27.38	25.95
26	32.53	30.95	29.63	28.15
28	34.88	33.24	31.87	30.34
30	37.21	35.52	34.11	32.53
35	42.98	41.18	39.66	37.96
40	48.70	46.78	45.17	43.35
45	54.37	52.35	50.65	48.73
50	60.01	57.88	56.10	54.08

P= 7 S=5

N \ α	.01	.025	.05	.10
8	15.74	14.49	13.46	12.34
9	17.33	16.02	14.95	13.77
10	18.87	17.51	16.40	15.18
11	20.38	18.98	17.82	16.55
12	21.86	20.41	19.22	17.91
13	23.32	21.83	20.60	19.25
14	24.76	23.23	21.97	20.57
15	26.18	24.61	23.32	21.88
16	27.58	25.98	24.65	23.18
17	28.97	27.34	25.98	24.47
18	30.35	28.68	27.29	25.75
19	31.72	30.01	28.60	27.02
20	33.07	31.34	29.89	28.29
22	35.76	33.96	32.46	30.79
24	38.41	36.55	35.00	33.27
26	41.03	39.11	37.51	35.73
28	43.62	41.65	40.01	38.17
30	46.20	44.17	42.48	40.59
35	52.55	50.40	48.60	46.53
40	58.80	56.54	54.64	52.51
45	64.98	62.60	60.61	58.37
50	71.08	68.61	66.53	64.19

TABLE 1 (CONTINUED)

P= 7 S=6

N	α .01	.025	.05	.10
8	23.70	21.93	20.48	18.90
9	25.52	23.70	22.20	20.57
10	27.29	25.42	23.88	22.20
11	29.02	27.10	25.52	23.73
12	30.71	28.74	27.13	25.35
13	32.37	30.36	28.71	26.83
14	34.01	31.95	30.27	28.40
15	35.62	33.52	31.80	29.90
16	37.20	35.07	33.32	31.38
17	38.77	36.60	34.81	32.84
18	40.32	38.12	36.30	34.28
19	41.86	39.62	37.77	35.72
20	43.38	41.10	39.22	37.14
22	46.38	44.04	42.10	39.95
24	49.33	46.92	44.93	42.72
26	52.24	49.78	47.73	45.46
28	55.12	52.59	50.50	48.17
30	57.97	55.38	53.24	50.85
35	64.98	62.24	59.98	57.46
40	71.81	68.96	66.60	63.96
45	78.54	75.58	73.11	70.36
50	85.18	82.10	79.54	76.63

P= 8 S=1

N	α .01	.025	.05	.10
9	1.15	.92	.75	.58
10	1.59	1.40	1.18	.95
11	2.25	1.91	1.64	1.37
12	2.83	2.44	2.14	1.91
13	3.43	3.00	2.66	2.29
14	4.05	3.58	3.20	2.78
15	4.69	4.17	3.75	3.30
16	5.34	4.78	4.33	3.84
17	6.00	5.41	4.92	4.39
18	6.67	6.04	5.52	4.95
19	7.35	6.69	6.14	5.53
20	8.04	7.34	6.77	6.13
22	9.45	8.68	8.05	7.34
24	10.88	10.05	9.37	8.60
26	12.34	11.45	10.71	9.88
28	13.82	12.88	12.09	11.20
30	15.32	14.32	13.48	12.54
35	19.15	18.02	17.07	15.99
40	23.05	21.81	20.75	19.55
45	27.03	25.67	24.52	23.20
50	31.07	29.60	28.35	25.93

TABLE 1 (CONTINUED)

P= 8 S=2

N	α .01	.025	.05	.10
9	2.90	2.50	2.19	1.85
10	3.69	3.24	2.88	2.50
11	4.48	3.99	3.58	3.15
12	5.29	4.74	4.30	3.83
13	6.10	5.51	5.03	4.51
14	6.92	6.29	5.78	5.22
15	7.74	7.08	6.53	5.93
16	8.57	7.87	7.29	6.65
17	9.41	8.67	8.06	7.39
18	10.25	9.48	8.84	8.13
19	11.10	10.29	9.62	8.88
20	11.95	11.11	10.41	9.64
22	13.66	12.76	12.01	11.17
24	15.38	14.42	13.62	12.73
26	17.12	16.10	15.25	14.30
28	18.86	17.79	16.90	15.89
30	20.62	19.50	18.56	17.50
35	25.04	23.80	22.75	21.58
40	29.50	28.15	27.01	25.72
45	34.00	32.54	31.31	29.92
50	38.52	36.97	35.66	34.16

P= 8 S=3

N	α .01	.025	.05	.10
9	5.40	4.82	4.36	3.86
10	6.41	5.79	5.28	4.73
11	7.42	6.74	6.20	5.60
12	8.42	7.70	7.11	6.47
13	9.42	8.66	8.04	7.35
14	10.41	9.62	8.96	8.24
15	11.41	10.57	9.89	9.12
16	12.40	11.53	10.81	10.01
17	13.40	12.49	11.74	10.91
18	14.39	13.45	12.67	11.81
19	15.39	14.41	13.61	12.71
20	16.38	15.38	14.54	13.61
22	18.36	17.30	16.42	15.43
24	20.35	19.23	18.29	17.25
26	22.33	21.16	20.18	19.08
28	24.32	23.09	22.07	20.91
30	26.30	25.03	23.96	22.76
35	31.27	29.87	28.70	27.38
40	36.23	34.73	33.47	32.04
45	41.19	39.59	38.24	36.71
50	46.16	44.46	43.03	41.41

TABLE 1 (CONTINUED)

P= 8 S=4

N	α .01	.025	.05	.10
9	8.76	7.99	7.35	6.66
10	9.98	9.16	8.48	7.74
11	11.19	10.31	9.60	8.81
12	12.38	11.46	10.71	9.87
13	13.55	12.60	11.81	10.93
14	14.72	13.72	12.90	11.93
15	15.88	14.85	13.99	13.04
16	17.03	15.96	15.08	14.09
17	18.17	17.07	16.16	15.14
18	19.31	18.18	17.24	16.19
19	20.45	19.28	18.31	17.23
20	21.58	20.38	19.39	18.27
22	23.83	22.57	21.53	20.35
24	26.07	24.75	23.66	22.43
26	28.29	26.92	25.78	24.50
28	30.50	29.09	27.90	26.57
30	32.71	31.24	30.02	28.63
35	38.19	36.61	35.28	33.79
40	43.64	41.95	40.53	38.93
45	49.05	47.27	45.76	44.06
50	54.45	52.56	50.97	49.18

P= 8 S=5

N	α .01	.025	.05	.10
9	13.21	12.18	11.34	10.41
10	14.63	13.55	12.67	11.70
11	16.02	14.90	13.98	12.96
12	17.39	16.23	15.27	14.21
13	18.74	17.54	16.55	15.45
14	20.08	18.84	17.81	16.67
15	21.40	20.12	19.06	17.89
16	22.70	21.39	20.30	19.09
17	24.00	22.65	21.53	20.29
18	25.28	23.90	22.76	21.48
19	26.56	25.15	23.98	22.67
20	27.82	26.38	25.19	23.85
22	30.34	28.84	27.59	26.19
24	32.82	31.27	29.97	28.52
26	35.29	33.68	32.33	30.83
28	37.73	36.07	34.68	33.13
30	40.16	38.45	37.02	35.41
35	46.17	44.34	42.81	41.08
40	52.10	50.16	48.54	46.71
45	57.97	55.93	54.22	52.29
50	63.79	61.66	59.86	57.84

TABLE 1 (CONTINUED)

P= 8 S =6

N	α .01	.025	.05	.10
9	19.14	17.78	16.67	15.44
10	20.76	19.36	18.20	16.93
11	22.35	20.90	19.71	18.39
12	23.90	22.41	21.18	19.82
13	25.43	23.90	22.63	21.23
14	26.93	25.37	24.07	22.63
15	28.42	26.81	25.48	24.01
16	29.88	28.24	26.88	25.37
17	31.33	29.66	28.27	26.72
18	32.77	31.06	29.64	28.05
19	34.19	32.45	31.00	29.39
20	35.60	33.83	32.35	30.71
22	38.38	36.55	35.03	33.32
24	41.13	39.24	37.67	35.91
26	43.85	41.90	40.28	38.46
28	46.53	44.53	42.86	41.00
30	49.19	47.14	45.43	43.51
35	55.75	53.58	51.76	49.72
40	62.19	59.90	57.99	55.84
45	68.54	66.15	64.14	61.89
50	74.81	72.32	70.23	67.88

P= 8 S=7

N	α .01	.025	.05	.10
9	27.56	25.68	24.14	22.46
10	29.40	27.47	25.89	24.16
11	31.19	29.22	27.60	25.82
12	32.95	30.93	29.27	27.45
13	34.67	32.61	30.92	29.05
14	36.36	34.26	32.53	30.63
15	38.02	35.88	34.13	32.18
16	39.66	37.49	35.70	33.72
17	41.28	39.07	37.25	35.23
18	42.88	40.64	38.78	36.73
19	44.46	42.18	40.30	38.22
20	46.03	43.72	41.81	39.69
22	49.12	46.74	44.77	42.59
24	52.16	49.72	47.70	45.46
26	55.15	52.65	50.58	48.28
28	58.11	55.55	53.43	51.07
30	61.03	58.42	56.25	53.84
35	68.20	65.46	63.18	60.64
40	75.21	72.35	69.97	67.31
45	82.10	79.12	76.64	73.87
50	88.89	85.80	83.23	80.35

TABLE 1 (CONTINUED)

P= 9 S =1

N \ α	.01	.025	.05	.10
10	1.02	.92	.67	.51
11	1.51	1.25	1.05	.85
12	2.02	1.71	1.48	1.23
13	2.55	2.20	1.93	1.63
14	3.11	2.72	2.40	2.07
15	3.68	3.25	2.90	2.53
16	4.27	3.80	3.42	3.01
17	4.87	4.37	3.95	3.50
18	5.49	4.95	4.50	4.02
19	6.12	5.54	5.07	4.55
20	6.75	6.15	5.65	5.09
22	8.06	7.39	6.83	6.21
24	9.40	8.67	8.06	7.38
26	10.77	9.98	9.32	8.58
28	12.17	11.32	10.61	9.82
30	13.59	12.69	11.93	11.08
35	17.22	16.19	15.33	14.35
40	20.95	19.81	18.84	17.74
45	24.76	23.51	22.44	21.23
50	28.64	27.28	26.13	24.81

P= 9 S=2

N \ α	.01	.025	.05	.10
10	2.57	2.22	1.94	1.65
11	3.29	2.89	2.57	2.23
12	4.01	3.57	3.21	2.82
13	4.75	4.26	3.87	3.44
14	5.50	4.97	4.54	4.07
15	6.26	5.69	5.23	4.72
16	7.02	6.42	5.93	5.38
17	7.80	7.16	6.64	6.06
18	8.58	7.91	7.35	6.74
19	9.36	8.66	8.08	7.44
20	10.16	9.42	8.81	8.14
22	11.76	10.97	10.30	9.57
24	13.38	12.53	11.82	11.03
26	15.02	14.11	13.36	12.51
28	16.67	15.71	14.91	14.01
30	18.33	17.33	16.48	15.53
35	22.54	21.42	20.47	19.41
40	26.81	25.58	24.54	23.36
45	31.13	29.79	28.66	27.39
50	35.48	34.05	32.84	31.46

TABLE 1 (CONTINUED)

P= 9 S=3

N	α .01	.025	.05	.10
10	4.76	4.26	3.85	3.41
11	5.58	5.13	4.68	4.20
12	6.40	6.01	5.52	4.99
13	7.53	6.89	6.37	5.79
14	8.45	7.77	7.22	6.60
15	9.37	8.66	8.07	7.42
16	10.30	9.55	8.93	8.25
17	11.22	10.44	9.79	9.08
18	12.15	11.34	10.66	9.91
19	13.08	12.24	11.53	10.75
20	14.01	13.14	12.41	11.60
22	15.88	14.95	14.17	13.30
24	17.75	16.76	15.94	15.01
26	19.63	18.59	17.72	16.74
28	21.51	20.42	19.50	18.47
30	23.40	22.26	21.30	20.22
35	28.13	26.87	25.82	24.63
40	32.87	31.52	30.37	29.08
45	37.64	36.18	34.95	33.56
50	42.41	40.86	39.55	38.07

P= 9 S=4

N	α .01	.025	.05	.10
10	7.68	7.00	6.45	5.84
11	8.79	8.07	7.47	6.82
12	9.89	9.13	8.50	7.80
13	10.98	10.18	9.51	8.78
14	12.07	11.23	10.53	9.76
15	13.15	12.27	11.54	10.74
16	14.22	13.31	12.55	11.71
17	15.30	14.35	13.56	12.69
18	16.37	15.39	14.57	13.67
19	17.43	16.42	15.58	14.64
20	18.49	17.46	16.59	15.62
22	20.61	19.52	18.60	17.58
24	22.73	21.58	20.61	19.54
26	24.83	23.63	22.62	21.49
28	26.93	25.68	24.63	23.45
30	29.03	27.73	26.64	25.41
35	34.26	32.85	31.66	30.32
40	39.46	37.95	36.67	35.23
45	44.65	43.04	41.68	40.15
50	49.83	48.13	46.70	45.07

TABLE 1 (CONTINUED)

P= 9 S=5

N	α .01	.025	.05	.10
10	11.44	10.56	9.84	9.05
11	12.73	11.81	11.05	10.22
12	14.01	13.05	12.25	11.37
13	15.27	14.26	13.44	12.52
14	16.51	15.47	14.61	13.65
15	17.74	16.57	15.78	14.79
16	18.97	17.86	16.94	15.91
17	20.18	19.04	18.09	17.03
18	21.38	20.21	19.24	18.15
19	22.58	21.38	20.38	19.26
20	23.78	22.54	21.52	20.37
22	26.14	24.85	23.78	22.57
24	28.49	27.15	26.03	24.77
26	30.82	29.43	28.26	26.95
28	33.14	31.70	30.49	29.13
30	35.45	33.95	32.70	31.30
35	41.16	39.56	38.21	36.63
40	46.82	45.11	43.68	42.05
45	52.44	50.63	49.12	47.40
50	58.02	56.12	54.53	52.72

P= 9 S=6

N	α .01	.025	.05	.10
10	16.27	15.14	14.22	13.20
11	17.74	16.57	15.61	14.55
12	19.19	17.98	16.98	15.88
13	20.61	19.36	18.33	17.19
14	22.01	20.73	19.67	18.49
15	23.39	22.08	20.99	19.77
16	24.76	23.41	22.29	21.04
17	26.12	24.74	23.59	22.31
18	27.46	26.05	24.87	23.56
19	28.80	27.35	26.15	24.80
20	30.12	28.64	27.41	26.04
22	32.74	31.20	29.93	28.50
24	35.32	33.74	32.41	30.93
26	37.88	36.25	34.87	33.34
28	40.42	38.73	37.32	35.73
30	42.94	41.20	39.75	38.11
35	49.15	47.30	45.75	44.00
40	55.27	53.32	51.68	49.83
45	61.32	59.27	57.54	55.60
50	67.31	65.17	63.36	61.32

TABLE I (CONTINUED)

P= 9 S =7

N	α .01	.025	.05	.10
10	22.59	21.14	19.94	18.62
11	24.24	22.75	21.52	20.15
12	25.86	24.33	23.06	21.65
13	27.45	25.88	24.58	23.14
14	29.02	27.41	26.08	24.60
15	30.56	28.92	27.55	26.04
16	32.08	30.40	29.01	27.46
17	33.59	31.87	30.45	28.87
18	35.07	33.33	31.88	30.26
19	36.54	34.77	33.29	31.64
20	38.00	36.20	34.69	33.02
22	40.88	39.02	37.46	35.73
24	43.72	41.80	40.20	38.41
26	46.52	44.54	42.90	41.05
28	49.29	47.26	45.57	43.67
30	52.03	49.95	48.22	46.27
35	58.77	56.57	54.74	52.68
40	65.38	63.08	61.15	58.98
45	71.89	69.48	67.47	65.20
50	78.31	75.81	73.71	71.35

P= 9 S=8

N	α .01	.025	.05	.10
10	31.43	29.45	27.83	26.05
11	33.28	31.26	29.61	27.78
12	35.09	33.03	31.34	29.47
13	36.87	34.77	33.04	31.13
14	38.61	36.47	34.71	32.76
15	40.32	38.15	36.35	34.37
16	42.01	39.80	37.98	35.96
17	43.68	41.43	39.58	37.52
18	45.33	43.05	41.16	39.07
19	46.95	44.64	42.73	40.60
20	48.56	46.22	44.27	42.12
22	51.74	49.33	47.33	45.12
24	54.85	52.38	50.34	48.06
26	57.92	55.40	53.30	50.97
28	60.95	58.37	56.23	53.84
30	63.94	61.31	59.12	56.68
35	71.28	68.52	66.22	63.65
40	78.45	75.57	73.17	70.49
45	85.48	82.49	79.99	77.21
50	92.40	89.30	86.72	83.83

TABLE 1 (CONTINUED)

P=10 S=1

N	α .01	.025	.05	.10
11	.92	.74	.60	.46
12	1.36	1.13	.95	.77
13	1.83	1.56	1.34	1.11
14	2.32	2.01	1.76	1.49
15	2.84	2.48	2.20	1.89
16	3.37	2.98	2.66	2.32
17	3.92	3.49	3.14	2.76
18	4.48	4.02	3.64	3.23
19	5.06	4.56	4.16	3.71
20	5.65	5.12	4.69	4.21
22	6.87	6.27	5.78	5.24
24	8.12	7.47	6.93	6.32
26	9.40	8.70	8.11	7.45
28	10.72	9.96	9.32	8.60
30	12.08	11.25	10.56	9.80
35	15.51	14.57	13.78	12.89
40	19.08	18.03	17.14	16.12
45	22.73	21.58	20.59	19.47
50	26.47	25.21	24.14	22.91

P=10 S=2

N	α .01	.025	.05	.10
11	2.31	1.99	1.75	1.48
12	2.96	2.60	2.32	2.01
13	3.63	3.23	2.91	2.56
14	4.32	3.87	3.52	3.13
15	5.01	4.53	4.14	3.71
16	5.72	5.20	4.78	4.32
17	6.43	5.88	5.43	4.93
18	7.16	6.57	6.09	5.56
19	7.89	7.27	6.77	6.21
20	8.63	7.98	7.45	6.86
22	10.13	9.43	8.84	8.19
24	11.65	10.90	10.26	9.56
26	13.20	12.39	11.71	10.95
28	14.77	13.90	13.18	12.37
30	16.35	15.43	14.67	13.81
35	20.36	19.33	18.47	17.50
40	24.45	23.31	22.36	21.28
45	28.59	27.36	26.32	25.14
50	32.78	31.46	30.34	29.06

TABLE 1 (CONTINUED)

P=10 S=3

N	α .01	.025	.05	.10
11	4.26	3.81	3.45	3.06
12	5.11	4.62	4.21	3.78
13	5.96	5.42	4.99	4.51
14	6.81	6.24	5.77	5.25
15	7.67	7.06	6.55	6.00
16	8.53	7.88	7.35	6.76
17	9.39	8.71	8.15	7.53
18	10.26	9.55	8.96	8.31
19	11.13	10.39	9.78	9.09
20	12.01	11.24	10.60	9.89
22	13.77	12.94	12.25	11.48
24	15.54	14.65	13.92	13.09
26	17.31	16.38	15.60	14.72
28	19.10	18.12	17.30	16.37
30	20.90	19.87	19.00	18.03
35	25.41	24.27	23.31	22.23
40	29.96	28.72	27.67	26.43
45	34.54	33.20	32.07	30.79
50	39.14	37.72	36.51	35.14

P=10 S=4

N	α .01	.025	.05	.10
11	6.84	6.24	5.75	5.21
12	7.86	7.22	6.69	6.11
13	8.88	8.20	7.63	7.02
14	9.89	9.17	8.58	7.92
15	10.90	10.14	9.52	8.83
16	11.91	11.12	10.46	9.74
17	12.91	12.09	11.41	10.65
18	13.92	13.06	12.35	11.56
19	14.92	14.04	13.30	12.48
20	15.92	15.01	14.25	13.40
22	17.92	16.96	16.15	15.24
24	19.92	18.90	18.05	17.09
26	21.92	20.85	19.96	18.95
28	23.92	22.80	21.86	20.81
30	25.92	24.75	23.78	22.67
35	30.91	29.64	28.57	27.36
40	35.90	34.53	33.37	32.06
45	40.89	39.43	38.19	36.78
50	45.88	44.33	43.01	41.52

TABLE I (CONTINUED)

P=10 S=5

N	α .01	.025	.05	.10
11	10.12	9.35	8.72	8.02
12	11.31	10.50	9.83	9.09
13	12.48	11.64	10.93	10.16
14	13.55	12.76	12.03	11.22
15	14.80	13.89	13.12	12.28
16	15.95	15.00	14.21	13.33
17	17.09	16.11	15.29	14.38
18	18.23	17.21	16.37	15.43
19	19.36	18.32	17.44	16.47
20	20.49	19.41	18.52	17.51
22	22.73	21.60	20.65	19.60
24	24.95	23.77	22.78	21.67
26	27.17	25.94	24.90	23.74
28	29.37	28.09	27.02	25.81
30	31.57	30.24	29.13	27.88
35	37.02	35.59	34.39	33.03
40	42.45	40.92	39.63	38.17
45	47.84	46.22	44.85	43.30
50	53.21	51.50	50.05	48.42

P=10 S=6

N	α .01	.025	.05	.10
11	14.23	13.26	12.47	11.59
12	15.58	14.58	13.75	12.82
13	16.91	15.87	15.01	14.05
14	18.23	17.15	16.26	15.26
15	19.53	18.42	17.49	16.46
16	20.82	19.67	18.72	17.66
17	22.09	20.92	19.93	18.84
18	23.36	22.15	21.14	20.02
19	24.61	23.38	22.35	21.19
20	25.86	24.60	23.54	22.36
22	28.34	27.02	25.91	24.68
24	30.79	29.42	28.27	26.98
26	33.22	31.80	30.60	29.26
28	35.63	34.16	32.92	31.54
30	38.02	36.51	35.23	33.80
35	43.95	42.32	40.96	39.42
40	49.80	48.08	46.63	44.99
45	55.60	53.78	52.25	50.52
50	61.35	59.44	57.84	56.02

TABLE 1 (CONTINUED)

P=10 S=7

N	α .01	.025	.05	.10
11	19.40	18.19	17.20	16.09
12	20.92	19.67	18.64	17.49
13	22.41	21.12	20.06	18.87
14	23.87	22.55	21.46	20.24
15	25.32	23.96	22.84	21.58
16	26.74	25.36	24.20	22.92
17	28.15	26.74	25.56	24.24
18	29.55	28.10	26.90	25.55
19	30.94	29.46	28.23	26.85
20	32.31	30.80	29.54	28.14
22	35.02	33.46	32.16	30.70
24	37.70	36.09	34.74	33.22
26	40.35	38.69	37.29	35.73
28	42.97	41.26	39.82	38.21
30	45.57	43.81	42.33	40.67
35	51.97	50.10	48.53	46.76
40	58.27	56.30	54.64	52.77
45	64.49	62.42	60.68	58.72
50	70.63	68.47	66.66	64.61

P=10 S=8

N	α .01	.025	.05	.10
11	26.08	24.54	23.27	21.87
12	27.76	26.18	24.88	23.44
13	29.41	27.80	26.46	24.98
14	31.03	29.38	28.02	26.50
15	32.62	30.94	29.55	27.99
16	34.20	32.48	31.06	29.47
17	35.75	34.00	32.55	30.93
18	37.29	35.51	34.03	32.38
19	38.80	37.00	35.49	33.81
20	40.31	38.47	36.94	35.23
22	43.28	41.38	39.80	38.03
24	46.20	44.25	42.62	40.80
26	49.06	47.07	45.40	43.53
28	51.92	49.87	48.15	46.23
30	54.73	52.63	50.87	48.91
35	61.65	59.43	57.58	55.49
40	68.42	66.10	64.15	61.97
45	75.08	72.66	70.63	68.34
50	81.64	79.13	77.02	74.64

TABLE 1 (CONTINUED)

P=10 S=9

N	α .01	.025	.05	.10
11	35.30	33.24	31.55	29.63
12	37.17	35.07	33.34	31.43
13	38.99	36.85	35.09	33.14
14	40.79	38.61	36.81	34.83
15	42.55	40.33	38.51	36.49
16	44.28	42.03	40.18	38.12
17	45.99	43.71	41.82	39.73
18	47.68	45.37	43.45	41.33
19	49.35	47.00	45.06	42.91
20	51.00	48.62	46.65	44.47
22	54.25	51.81	49.79	47.54
24	57.45	54.95	52.87	50.57
26	60.59	58.03	55.91	53.55
28	63.69	61.07	58.91	56.43
30	66.74	64.08	61.87	59.40
35	74.23	71.45	69.13	66.54
40	81.55	78.64	76.23	73.53
45	88.72	85.70	83.19	80.39
50	95.76	92.65	90.05	87.14